Condensate Phase Microscopy

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Outline:

- Phase retrieval algorithm
- Retrieval of a condensate phase from a TOF image
- Example: reconstruction of space domains in optical lattice systems

Phase retrieval algorithm

How to retrieve an object $\psi(\mathbf{r})$?

Assume that in 2D:

- modulus of the Fourier transform $\mathbf{M} = |\tilde{\psi}(\mathbf{k})|$ is known,
- phase in the **k** space is lost,
- support **S** of the object in the **r** space can be estimated.

How to retrieve the object $\psi(\mathbf{r})$?

$$|\tilde{\psi}(\mathbf{k})|^2 \propto \left| \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} \psi(\mathbf{r}_i) \right|^2$$

S. Marchesini, Rev. Sci. Instrum. **78**, 011301 (2007).

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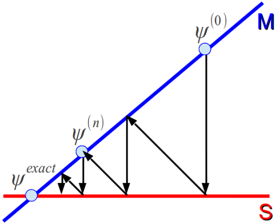
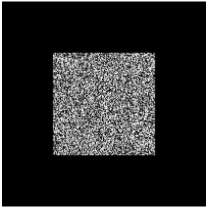
Phase retrieval algorithm



Phase retrieval algorithm

Original object $\psi(\mathbf{r})$

$\psi^{(1)}(\mathbf{r})$



The support S and $\left|\tilde{\psi}(\mathbf{k})\right|$ are known only.

- we start with random phases $\left|\tilde{\psi}(\mathbf{k})\right| e^{i\tilde{\phi}^{(0)}(\mathbf{k})}$,
- inverse Fourier transform and projection on the support S,
- Fourier transform and exchange $\left|\tilde{\psi}^{(1)}(\mathbf{k})\right| \rightarrow \left|\tilde{\psi}(\mathbf{k})\right|$.

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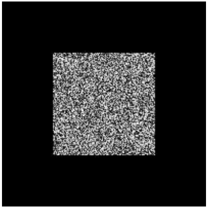
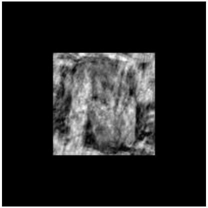


$\psi^{(3)}(\mathbf{r})$



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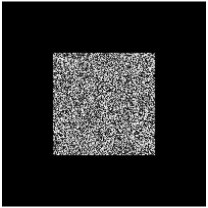

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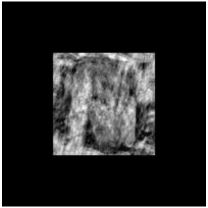
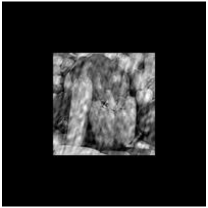


$\psi^{(20)}(\mathbf{r})$



Phase retrieval algorithm

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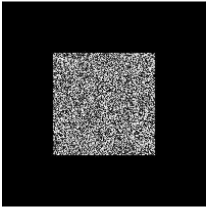

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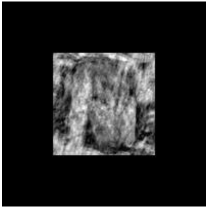

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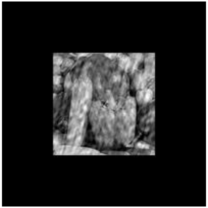

$\psi^{(50)}(\mathbf{r})$

BEC in an optical lattice in 2D

Time-Of-Flight images (in the far field limit):

$$I(\mathbf{r}) \propto |\tilde{\psi}(\mathbf{k})|^2 \propto \left| \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} \psi(\mathbf{r}_i) \right|^2, \quad \mathbf{k} = \frac{m\mathbf{r}}{\hbar\, t_{TOF}}.$$

We need to retrieve $\psi(\mathbf{r})$.

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At low temperature one can estimate:

$$|\psi(\mathbf{r})|^2 \approx |\varphi_{TF}(\mathbf{r})|^2 \sum_i |w_0(\mathbf{r} - \mathbf{r}_i)|^2.$$

Then, the projection on the density $|\psi(\mathbf{r})|^2$ can substitute for the projection on the support S.

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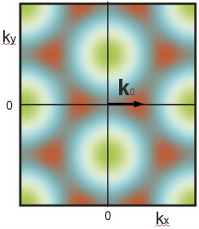
In the near field [F. Gerbier et al., PRL **101**, 155303 (2008)]:

$$I(\mathbf{r}) \propto |\tilde{\psi}(\mathbf{k})|^2 \propto \left| \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} \psi(\mathbf{r}_i) e^{-i\beta\mathbf{r}_i^2} \right|^2, \quad \beta = \frac{m}{2\hbar t_{TOF}},$$

then $\psi^{(n)}(\mathbf{r}) \to e^{i\beta\mathbf{r}^2}\psi^{(n)}(\mathbf{r})$.

BEC in a triangular optical lattice with negative tunneling amplitudes

Dispersion relation



Degenerate ground state:

$$\psi_{\pm \mathbf{k}_0}(\mathbf{r}) = \varphi_{TF}(\mathbf{r}) \sum_i e^{\pm i \mathbf{k}_0 \mathbf{r}_i} w_0(\mathbf{r} - \mathbf{r}_i)$$

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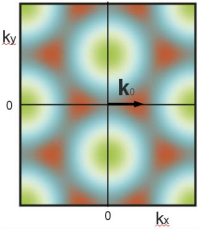

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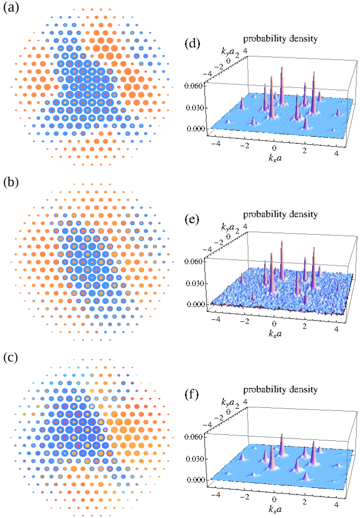


ideal image

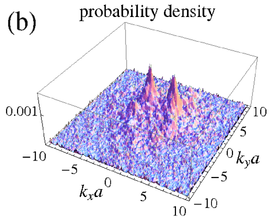
$$\frac{A}{\sigma n} = 0.5$$

$$\frac{\sigma_r}{\sigma_{peak}} = 0.1$$

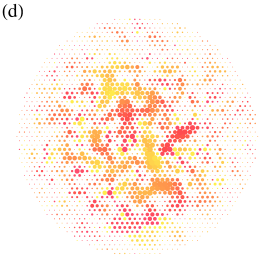
Analysis of the Hamburg experiment results

J. Struck, C. Ölschläger, R. Le Targat, P. Soltan-Panahi, A. Eckardt, M. Lewenstein, P. Windpassinger, K. Sengstock, Science **333**, 996 (2011).

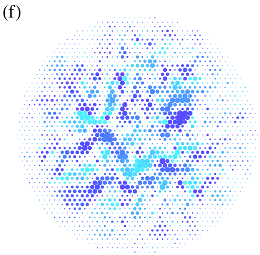
TOF image:



$\psi_{+\mathbf{k}_0}(\mathbf{r})$ domains:



$\psi_{-\mathbf{k}_0}(\mathbf{r})$ domains:



Conclusions

- Knowledge of a TOF image and estimate of the initial atomic density in an optical lattice potential are sufficient to retrieve phase of a BEC wave-function.
- Condensate phase microscopy is very useful when the order parameter of an ultra-cold atomic gas is complex, e.g., in the presence of artificial gauge potentials or in a multi-orbital superfluid phase in optical lattices.
- An example has been analyzed where the phase microscopy allows for reconstruction of a domain structure of a BEC in a triangular optical lattice.

A. Kosior and KS, Phys. Rev. Lett. **112**, 045302 (2014).