Emergence of nonlinearity in bosonic ultracold gases

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The advent of Bose-Einstein condensation and the possibility of creating experimentally ultracold quantum-degenerate atomic gases opened new and fascinating perspectives for the study of weakly interacting quantum systems. In the case of ultracold bosonic gases, the possibility of generating a mesoscopic quantum-coherent state is particularly interesting. At very low – yet experimentally accessible – temperatures, such systems can be described with very good accuracy by a nonlinear version of the Schrödinger equation, namely the mean-field Gross-Pitaevskii equation, in which a nonlinear term accounts for coherent particle-particle interactions. This means that such systems can in principle display "quasiclasssical" chaos, i.e., chaotic dynamics associated to exponential sensitivity to initial conditions, which is forbidden by the usual (linear) Schrödinger equation. One thus faces a paradox: The exact description of a boson gas corresponds to a (very complex) many-body problem which can be reduced to a system of coupled *linear* equations, and thus *cannot* display quasiclassical chaos! In going from the many-body formulation to the mean-field approximation, quasiclassical behavior has emerged, i.e., symmetries of the many body system have been broken. In the present work, we use a toy model consisting of a Bose-Einstein condensate confined to three adjacent sites of a tilted lattice, which constitutes a "minimal" system presenting quasiclassical chaos. We compare the dynamics of such a system obtained by numerical integration of the many-body problem with that observed by numerical integration of the Gross-Pitaevskii equation. This sheds some light on the emergence of the nonlinearity, and hints for possible explanations of the above-mentioned paradox.

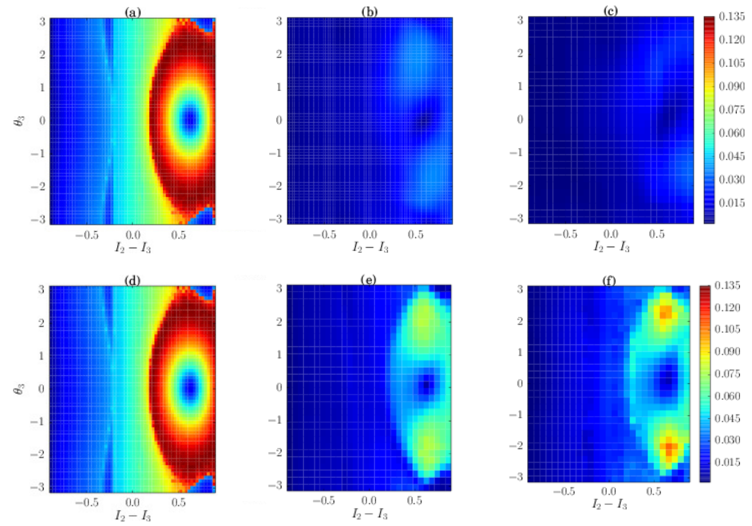

Figure 1: *Variance of the wave packet position calculated according to various methods: Gross-Pitaevskii equation (left column), Lanczos diagonalization (center column), and truncated Husimi representation (right column), and with different number of particles, 30 (top row) and 400 (bottom row). The red areas correspond to quasiclassical chaos, easily spotted in the Gross-Pitaevskii simulation, but clearly depending on the particle number for many-body approaches.*