

The self-energy of bound electron

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In present time accuracy of spectroscopic measurements achieved fantastic accuracy 1 part in 10 to the power 14. If these measurements are followed by theoretical calculations of similar accuracy we can test quantum electrodynamics and part of the standard model responsible for parity violating weak forces by comparing theory and experiment and set some of the basic physical constants like Rydberg constant, mass and radius of nucleus, constant of fine structure etc. For achieving this kind of accuracy of calculations one must take into account the so called radiation corrections and in particular the most important of them, the self-energy of bound electron. Our method is based on one single assumption, namely that except for the part of the time component of the electron four-momentum corresponding to the electron rest mass, the exchange of four-momentum between the virtual electron and photon can be treated perturbatively. This assumption holds very well, except for the electron virtual states with very high three-momentum. It can be shown that for such virtual states one can always rearrange the pertinent expression in a way that allows the electron to be treated as free. The fraction of the free-particle approximation contained in the relativistic multipole expansion carried out to a given order can be precisely determined. By taking the method up to the ninth order and estimating the remainder of the series, the result obtained for the ground state of the hydrogen atom differs from the other result of comparable accuracy by 2 parts in 10⁹. This amounts to the difference of 18 Hz for 2s – 1s transition in hydrogen. This is by four orders smaller than the uncertainty in determination of the proton radius.

For years the self-energy for the atoms with low nuclear charges has been calculated via the series in powers of $Z\alpha$

$$E = \frac{m\alpha(Z\alpha)^4}{\pi n^3 s^3} F(Z\alpha, n, l_j). \quad (1)$$

The more recent methods used to calculate self-energy are based on the partial wave expansion. However, for atoms with low nuclear charges this method converges very slowly. Nonetheless, the most accurate results obtained so far for the hydrogen atom were obtained in [1] by means of this expansion. In that paper several millions of partial waves were considered. For each partial wave there is a three-dimensional integration to be performed numerically

Our method is based on the assumption that the four-momentum of the bound electron in virtual states is dominated by the four-momentum of the electron at rest:

$$\frac{1}{k^2 - 2k \cdot \varepsilon + H - 2k \cdot (\quad - \varepsilon)} = \frac{1}{z - \tilde{H}_0 - \lambda \tilde{H}_1}, \quad (2)$$

where ε is the four-momentum of electron at rest, k is the four-momentum of photon and \quad is the four-momentum of virtual electron. Only few terms have to be taken in order to get the desired accuracy. The only integrals to be performed numerically are one-dimensional integrals over the electron wave numbers of the continuous part of the spectrum. Taking the method to the ninth order we obtain results

Term	$F(\alpha, 1, 0)$
Lead	10.315870916
Total	10.316793675(50)
Other	10.316793650(1)

Table 1: Lead order stands for the first two terms in the expansion. Total is the method taken to the ninth order. Other stands for the result taken from [1].

References

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