Multipole expansions in the theory of light radiation by atomic systems

M.Ya. Agre[1]

[1] *Department of Physical and Mathematical Sciences, National University of "Kyiv-Monyla Academy", 04655 Kyiv, Ukraine*

Presenting Author: magrik@ukr.net

The probability of spontaneous radiation of a photon with the unit polarization vector $\mathbf{e}$ and the momentum $\mathbf{k}$ is written down through the so-called radiation amplitude

$$V_{fi} = \mathbf{e}^* \int e^{-i\mathbf{kr}}\, \mathbf{j}_{fi}(\mathbf{r})\, d\mathbf{r}, \tag{1}$$

where $\mathbf{j}_{fi}$ is the current density vector for the transition of a radiating system from the initial to final state [1]. The cross section of photoeffect and the composite matrix elements for multiphoton transitions are also expressed through V_{fi} (1). In the first nonvanishing order of long-wave approximation ($ka << 1$, where a is the radius of the atomic system) V_{fi} proves to be proportional to the electric dipole moment of transition that corresponds to dipole approximation. In general case multipole expansion of the radiation amplitude (1) is very useful, but the structure of the known expansion makes it difficult for applications.

In the present paper we derive the compact multipole expansion for the amplitude of spontaneous radiation (1) using the mathematical technique of irreducible tensors [2]. The found multipole series gives to the total amplitude V_{fi} the form of the sum of amplitudes

$$V_{fi} = \sum_{l=1}^{\infty} \left(V_l^{(E)} + V_l^{(M)} \right), \tag{2}$$

where $V_l^{(E)}$ is the radiation amplitude of electric 2^l-pole (El) photon and $V_l^{(M)}$ is the radiatiion amplitude of magnetic 2^l-pole (Ml) photon. Here

$$V_l^{(E)} = \left(D_l \cdot \{\mathbf{e}^* \otimes Y_{l-1}(\mathbf{k}_0)\}_l \right), \qquad V_l^{(M)} = \left(A_l^{(l)} \cdot \{\mathbf{e}^* \otimes Y_l(\mathbf{k}_0)\}_l \right), \tag{3}$$

where $Y_{lm}(\mathbf{k}_0)$ is the spherical function, the unit vector $\mathbf{k}_0 = \mathbf{k}/k$, and all information about the radiating system is contained in the coefficients of the series (2), which are the irreducible tensors determined by the current density of transition,

$$A_{Lm}^{(l)} = 4\pi(-i)^l \int \left\{ g_l(kr) Y_l(\mathbf{n}) \otimes \mathbf{j}_{fi}(\mathbf{r}) \right\}_{Lm} d\mathbf{r}, \qquad D_{lm} = A_{lm}^{(l-1)} + \sqrt{\frac{l}{l+1}} A_{lm}^{(l+1)}, \tag{4}$$

where $g_l(x) = \sqrt{\pi/(2x)} J_{l+1/2}(x)$ is the spherical Bessel function, $\mathbf{n} = \mathbf{r}/r$. The standard designations [2] for the tensor and scalar products of two irreducible tensors are used in equations (3) and (4).

The angular momentum and parity selection rules keep only few terms in the series (2). In the long-wave approximation $A_{Lm}^{(l)}$ (4) has the order $(ka)^l$, $A_{lm}^{(l)}$ proving to be proportional to magnetic 2^l-pole moment of transition and $A_{lm}^{(l-1)}$, D_{lm} – to electric one. Correspondingly, $V_1^{(E)}$ in the series (2) becomes the amplitude of dipole radiation, $V_2^{(E)}$ becomes the amplitude of quadrupole radiation, $V_1^{(M)}$ becomes the amplitude of magnetic dipole radiation and so on.

The derived multipole expansion can be used in theoretical studies of electromagnetic field – atomic system interaction both in long-wave approximation and outside its framework.

References

[1] V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii, *Quantum Electrodynamics* (Pergamon Press, Oxford, 1982)

[2] D. A. Varshalovich, A. N. Moskalev, V. K. Khersonskii, *Quantum Theory of Angular Momentum* (World Sci. Publ., Singapore, 1988)