The M1-to-E2 and E1-to-M2 cross-susceptibilities of the Dirac one-electron atom

P. Stefańska[1] and R. Szmytkowski[1]

[1] *Atomic Physics Division, Department of Atomic, Molecular and Optical Physics, Faculty of Applied Physics and Mathematics, Gdańsk University of Technology, Narutowicza 11/12, 80–233 Gdańsk, Poland*

Presenting Author: pstefanska@mif.pg.gda.pl

We consider a Dirac one-electron atom placed in a weak, static, uniform magnetic $\boldsymbol{B}$ [or electric $\boldsymbol{F}$] field. We show that, to the first order in the strength of the perturbing field, the only electric $\boldsymbol{\mathcal{Q}}^{(1)}$ [or magnetic $\boldsymbol{\mathcal{M}}^{(1)}$] multipole moment induced by the field in the ground state of the atom is the quadrupole one. The coordinate-free form of these tensors are respectively

$$\boldsymbol{\mathcal{Q}}_{2\mu}^{(1)} = (4\pi\epsilon_0)\, c\, \alpha_{M1\to E2} \left[\frac{3}{4}(\boldsymbol{\nu}_\mu \boldsymbol{B} + \boldsymbol{B}\boldsymbol{\nu}_\mu) - \frac{1}{2}(\boldsymbol{\nu}_\mu \cdot \boldsymbol{B})\mathcal{I}\right] \tag{1}$$

and

$$\boldsymbol{\mathcal{M}}_{2}^{(1)} = (4\pi\epsilon_0)\, c\, \alpha_{E1\to M2} \left[\frac{3}{4}(\boldsymbol{\nu} \boldsymbol{F} + \boldsymbol{F}\boldsymbol{\nu}) - \frac{1}{2}(\boldsymbol{\nu} \cdot \boldsymbol{F})\mathcal{I}\right], \tag{2}$$

where $\mathcal{I}$ is the unit dyad, $\boldsymbol{\nu}_\mu$ is the unit vector parallel (when $\mu = +1/2$) or antiparallel (when $\mu = -1/2$) to the field vector $\boldsymbol{B}$, while $\boldsymbol{\nu}$ is the unit vector antiparallel to the permanent magnetic dipole moment of the atom.

The coefficients $\alpha_{M1\to E2}$ and $\alpha_{E1\to M2}$ appearing above are the magnetic-dipole-to-electric-quadrupole and the electric-dipole-to-magnetic-quadrupole cross-susceptibilities of the atom, respectively. Using the Sturmian expansion of the generalized Dirac–Coulomb Green function [1], we derive closed-form expressions for these two quantities. The results are of the form

$$\alpha_{M1\to E2} = \frac{\alpha a_0^4}{Z^4}\frac{\Gamma(2\gamma_1+5)}{720\Gamma(2\gamma_1)}\left[(\gamma_1+1)\mathcal{R}-1\right] \text{ and } \alpha_{E1\to M2} = -\frac{\alpha a_0^4}{Z^4}\frac{\Gamma(2\gamma_1+5)}{240\Gamma(2\gamma_1)}\left[(\gamma_1-2)\mathcal{R}-1\right], \tag{3}$$

where we have defined

$$\mathcal{R} = \frac{(\gamma_1+\gamma_2)\Gamma(\gamma_1+\gamma_2+2)\Gamma(\gamma_1+\gamma_2+3)}{\gamma_1\Gamma(2\gamma_1+5)\Gamma(2\gamma_2+1)}\, {}_3F_2\left(\begin{matrix}\gamma_2-\gamma_1-2,\ \gamma_2-\gamma_1-1,\ \gamma_2-\gamma_1 \\ \gamma_2-\gamma_1+1,\ 2\gamma_2+1\end{matrix};1\right). \tag{4}$$

Here $\Gamma(z)$ is the Euler's gamma function, $\gamma_\kappa = \sqrt{\kappa^2-(\alpha Z)^2}$, α denotes the Sommerfeld fine structure constant, while ${}_3F_2$ is the generalized hypergeometric function.

In the nonrelativistic limit, $\alpha_{M1\to E2}$ tends to zero. This agrees with earlier calculations [4–6] of that quantity, based on the Schrödinger or Pauli equation for the electron, which predicted the *quadratic* dependence of the induced electric quadrupole moment on the magnetic induction B.

The first of the authors very recently received a more general result for $\alpha_{M1\to E2}$, which describes this quantity for an arbitrary excited state of the atom. [7]

References

[1] R. Szmytkowski J. Phys. B **30**, 825–861 (1997) [erratum: **30**, 2747 (1997)]
[2] R. Szmytkowski, P. Stefańska Phys. Rev. A **85**, 042502/1–6 (2012)
[3] R. Szmytkowski, P. Stefańska Phys. Rev. A **89**, 012501/1–7 (2014)
[4] C. A. Coulson, M. J. Stephen Proc. Phys. Soc. A **69**, 777–782 (1956)
[5] A. V. Turbiner Yad. Fiz. **46**, 204–218 (1987)
[6] A. Y. Potekhin, A. V. Turbiner Phys. Rev. A **63**, 065402/1–4 (2001)
[7] P. Stefańska, (PhD Thesis, Gdańsk University of Technology, 2014)